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Reg. No. [| | | | | | | |]

I Semester B.Sc. Degree Examination, August - 2021
MATHEMATICS-I
(CBCS Semester Scheme Repeater)

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates : Answer **All** questions.

PART - A

Answer any **FIVE** questions. **(5×2=10)**

1. a) Define rank of the Matrix.

   b) Find the characteristic equation of the matrix $\begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$.

   c) Find the n[th] derivative of $\log(2x+3)$.

   d) If $u=x^2y^3$, find $\frac{\partial^2 u}{\partial x \partial y}$.

   e) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^6 x\, dx$.

   f) Evaluate $\int_{0}^{\frac{\pi}{2}} \sin^4 x \cos^6 x\, dx$.

   g) Find the equation of the sphere with centre at $(1,0,-2)$ and radius 2 units.

   h) Find the angle between the line $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z+4}{3}$ and the plane $2x+3y-z-4=0$.

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PART - B

Answer **One** full question. **(1×15=15)**

2. a) Find the rank of the matrix $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{bmatrix}$ by reducing to row reduced echelon form.

b) Solve completely the following system of equations $x+y=0$, $x-y-z=0$, $3x+y-z=0$.

c) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

(OR)

3. a) Reduce the matrix $\begin{bmatrix} 1 & 2 & 4 \\ -1 & -2 & -4 \\ 2 & 4 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ into normal form.

b) Verify the following system of equations for consistency and if consistant solve $x+y+z=4$, $2x+y-z=1$ and $x-y+2z=2$.

c) Verify Cayley - Hamilton theorem for the matrix $\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$.

PART - C

Answer **TWO** full questions. **(2×15=30)**

4. a) Find the nth derivative of $e^{ax}\cos(bx+c)$.

b) Find the nth derivative of $\frac{1}{6x^2-5x+1}$.

c) If $y=\sin^{-1}x$ show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$.

(OR)

5. a) If $u=\phi(y+ax)+\psi(y-ax)$ show that $\frac{\partial^2 u}{\partial x^2}=a^2\frac{\partial^2 u}{\partial y^2}$.

b) State and prove the Euler's theorem.

c) If $u=(x-y)^2+(y-z)^2+(z-x)^2$ show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.

6. a) If $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$, $z=r\cos\theta$ show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}=r^2\sin\theta$.

b) If $x=u(1-v)$, $y=uv$ find $J=\frac{\partial(x,y)}{\partial(u,v)}$ and $J'=\frac{\partial(u,v)}{\partial(x,y)}$. Also verify $J.J'=1$.

c) Obtain the reduction formula for $\int \tan^n x\, dx$.

(OR)

7. a) Evaluate $\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{3}{2}}dx$.

b) Show that $\int_0^{\pi} x\sin^3 x\, dx = \frac{2\pi}{3}$.

c) Evaluate $\int_0^1 \frac{x^{\alpha}-1}{\log x}dx$ using Leibnitz's rule of differentiation under integral sign.

PART - D

Answer **One** full question. **(1×15=15)**

8. a) Find the equation of the plane through the intersection of the planes $x-2y+z-7=0$ and $2x+3y-4z=0$ and cutting intercept 4 units on the x - axis.

b) Find the equation of the right circular cone whose vertex is $(1,-1,2)$, axis along the line $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-2}{-2}$ and semi vertical angle 45^0.

c) Find the equation of the sphere in vector form whose centre is at $2i-3j-4k$ and radius equal to 5 units.

(OR)

9. a) Find the length and the equation of the line of shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$.

b) Derive the equation of the right circular cone in its standard form $x^2+y^2=z^2\tan^2\alpha$.

c) Find the equation of the right circular cylinder whose generators touch the sphere $x^2+y^2+z^2=9$ and are parallel to the line $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-3}{5}$.